NAMIBIA UNIVERSITY
OF SCIENCE AND TECHNOLOGY

FACULTY OF HEALTH AND APPLIED SCIENCES

DEPARTMENT OF NATURAL AND APPLIED SCIENCES

QUALIFICATION : BACHELOR OF SCIENCE	
QUALIFICATION CODE: 07BOSC	**LEVEL:** 7
COURSE NAME: QUANTUM PHYSICS	**COURSE CODE:** QPH 702S
SESSION: NOVEMBER 2019	**PAPER:** THEORY
DURATION: 3 HOURS	**MARKS:** 100

FIRST OPPORTUNITY EXAMINATION QUESTION PAPER	
EXAMINER(S)	**Prof Dipti R. Sahu**
MODERATOR:	**Dr Habatwa V. Mweene**

INSTRUCTIONS

1. Answer any **five** questions.
2. Write clearly and neatly.
3. Number the answers clearly.

PERMISSIBLE MATERIALS

Non-programmable Calculators

THIS QUESTION PAPER CONSISTS OF 3 PAGES (Including this front page)

Question 1 **[20]**

Consider a particle whose normalized wave function is

$$\psi(x) = 2\alpha\sqrt{\alpha}xe^{-\alpha x} \qquad x > 0$$
$$= 0 \qquad x < 0$$

(a) For what value of *x* does $P(x) = |\Psi(x)|^2$ peak? (5)
(b) Calculate $\langle x \rangle$ and $\langle x^2 \rangle$. (10)
(c) What is the probability that the particle is found between $x = 0$ and $x = 1/\alpha$? (5)

Question 2 **[20]**

2.1 State, giving your reasons, which of the following functions would make satisfactory Wave functions for all values of the variable *x*:

(i) Ne^{ax^2}

(ii) Ne^{-ax^2}

(iii) $Ne^{-ax^2}/(3-x)$

(iv) Ne^{-ax}

where *N* and *a* are constants. (5)

2.2 In a region of space, a particle with mass *m* and with zero energy has a time independent

wave function

$$\psi(x) = Axe^{-x^2/L^2}$$

where *A* and *L* are constants. Determine the potential energy *U(x)* of the particle. (10)

2.3 Write the expression $\langle \Psi | \Psi \rangle = 1$ as an explicit integral equation in three dimensions, assuming that $|\Psi\rangle$ represents a wave function $\Psi(r)$. Suppose you have $|\Psi\rangle = \sum_n Cn\,|n\rangle$ (where n in Cn as a subscript) where the $\{|n\rangle\}$ is a complete set of orthonormal states. What condition does the above equation impose on the C_n? (5)

Question 3 **[20]**

3.1 Suppose that the operator corresponding to some observable is called *Q*. List two properties of this operator and/or of its eigenfunctions $|n\rangle$. The latter satisfy the equation $Q|n\rangle = q_n|n\rangle$. Suppose further that the quantum-mechanical state of a system is given by

$$|\psi\rangle = \sum_n c_n |n\rangle$$

with several of the expansion coefficients being non-zero ($C_n \neq 0$). If you were to make a *single measurement* of the observable *Q*, what would you get as a result? (5)

3.2 The potential function for a problem is defined by:

$$V(x) = \begin{cases} 0; & -a < x < a \\ \infty; & |x| > a \end{cases}$$

(a) Sketch the potential *V(x)* (2)
(b) Find the solutions of the time-independent Schroedinger equation in the different regions (5)
(c) Interpret the results. (8)

Question 4 **[20]**

4.1 Find the probability that the electron in the ground-state of the H atom is less than a distance *a* from the nucleus. (5)

4.2 Which pairs of operators commute in the set L^2, L_x, L_y and L_z? How is this related to which quantities can be simultaneously determined with arbitrary precision? (5)

4. 3 Evaluate the following commutators and state the consequences of the results.
(i) $[x, p_x]$ (ii) $[y, p_z]$, where the symbols have their usual meanings. (10)

Question 5 **[20]**

5.1 Show explicitly that $S^2 = \hbar^2 s(s+1)$ (5)

5.2 Evaluate the matrix of L_x for $l = 1$. (5)

5.3 Show that for $|s\rangle = \cos(\vartheta/2)\,|\uparrow\rangle + \exp(i\varphi)\sin(\vartheta/2)\,|\downarrow\rangle$, we obtain

$\langle s | \hat{\sigma} | s = \mathbf{i}\sin\theta\cos\phi + \mathbf{j}\sin\theta\sin\phi + \mathbf{k}\cos\theta$ (10)

Question 6 **[20]**

6.1 A particle moves in the 1-dimensional potential $V(x) = \infty$, $|x| > a$, $V(x) = V_0\cos(\pi x/2a)$, $|x| \leq a$, Calculate the ground-state energy to first order in perturbation theory. The unperturbed system energy and wave function is given by (10)

$$E^{(n)} = \frac{\pi^2\hbar^2 n^2}{8ma^2}, \quad u^{(n)} = \frac{1}{\sqrt{a}}\left\{ \begin{matrix} \cos \\ \sin \end{matrix} \right\} \frac{n\pi x}{2a}; \quad n \left\{ \begin{matrix} \text{odd} \\ \text{even} \end{matrix} \right\}$$

6.2 Consider a charged particle in the 1D harmonic oscillator potential. Suppose the particle is placed in a weak, uniform electric field. Treat the electric field as a small perturbation and obtain the first order corrections to the harmonic oscillator energy eigenvalues. (10)

Useful Standard Integrals

$$\int_0^\infty x^n e^{-x} dx = n!$$

$$\int_{-\infty}^{\infty} e^{-y^2} dy = \sqrt{\pi}$$

$$\int_{-\infty}^{\infty} y^n e^{-y^2} dy = \frac{\sqrt{\pi}}{n}; \; n \text{ even} \qquad 0; \; n \text{ odd}$$

$$\int_{-\infty}^{\infty} e^{-\alpha y^2} e^{-\beta y} dy = \left(\frac{\pi}{\alpha}\right)^{\frac{1}{2}} e^{\frac{\beta^2}{4\alpha}}$$

-------------END----------